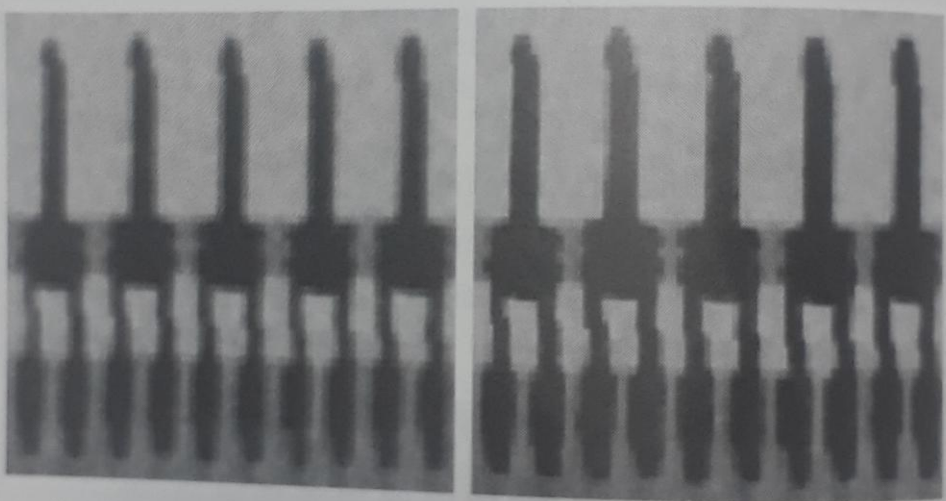
HW4 due July 15, class time

1.

the subimage on the left is the result of using an arithmetic mean filter of size 3×3; the other subimage is the result of using a geometric mean filter of the same size.

★(a) Explain why the subimage obtained with geometric mean filtering is less blurred. (*Hint:* Start your analysis by examining a 1-D step transition in intensity.)

(b) Explain why the black components in the right image are thicker.

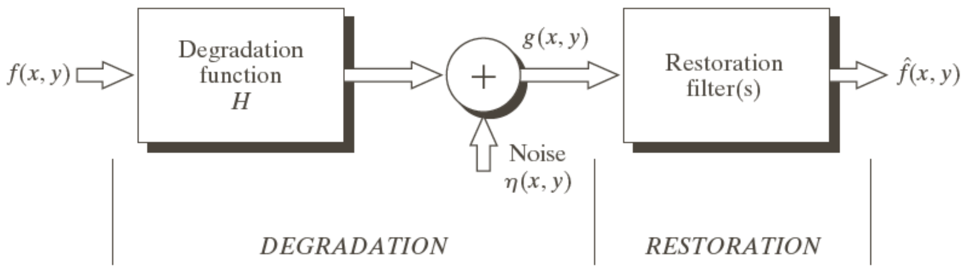


2.

Assume that the model in Fig. 5.1 is linear and position invariant and that the noise and image are uncorrelated. Show that the power spectrum of the output is

$$|G(u, v)|^2 = |H(u, v)|^2 |F(u, v)|^2 + |N(u, v)|^2$$

FIGURE 5.1 A model of the image degradation/restoration process.



3. Compute the 2D wavelet transform of the following image using Haar wavelets.

$$\mathbf{F} = \begin{bmatrix} 3 & -1 \\ 6 & 2 \end{bmatrix}$$

4.

Compute the expansion coefficients of 2-tuple $[3, 2]^T$ for the following bases and write the corresponding expansions:

★(a) Basis $\varphi_0 = \left[1/\sqrt{2}, 1/\sqrt{2},\right]^T$ and $\varphi_1 = \left[1/\sqrt{2}, -1/\sqrt{2},\right]^T$ on $\mathbf{R}^2$, the set of real 2-tuples.

(b) Basis $\varphi_0 = [1, 0]^T$ and $\varphi_1 = [1, 1]^T$, and its dual, $\widetilde{\varphi}_0 = [1, -1]^T$ and $\widetilde{\varphi}_1 = [0, 1]^T$, on $\mathbf{R}^2$.

(c) Basis $\varphi_0 = [1, 0]^T$, $\varphi_1 = \left[-1/2, \sqrt{3}/2\right]^T$, and $\varphi_2 = \left[-1/2, -\sqrt{3}/2\right]^T$, and their duals, $\widetilde{\varphi}_i = 2\varphi_i/3$ for $i = \{0, 1, 2,\}$, on $\mathbf{R}^2$.

5.

Suppose function $f(x)$ is a member of Haar scaling space V_3—that is, $f(x) \in V_3$. Use Eq. (7.2-22) to express V_3 as a function of scaling space V_0 and any required wavelet spaces. If $f(x)$ is 0 outside the interval $[0, 1)$, sketch the scaling and wavelet functions required for a linear expansion of $f(x)$ based on your expression.