HW 5 and 6

Due July 29, 18:00, drop in my mailbox

1.

Calculate and sketch the following

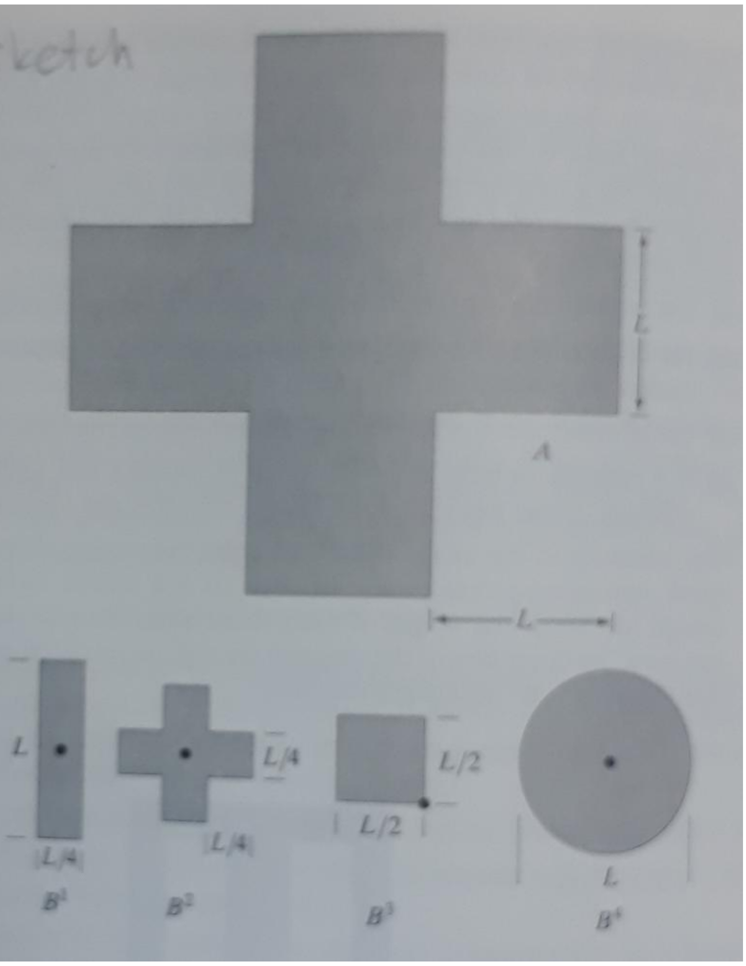
a) $(A \ominus B^4) \oplus B^2$

b) $(A \ominus B^1) \oplus B^3$

c) $(A \oplus B^1) \oplus B^3$

d) $(A \oplus B^3) \ominus B^2$

• represents the origin of operators.



2.

Prove the validity of the following expressions:

(a) $A \circ B$ is a subset (subimage) of A.

(b) If C is a subset of D, then $C \circ B$ is a subset of $D \circ B$.

(c) $(A \circ B) \circ B = A \circ B$.

3.

A binary image contains straight lines oriented horizontally, vertically, at 45°, and at −45°. Give a set of 3 × 3 masks that can be used to detect 1-pixel breaks in these lines. Assume that the intensities of the lines and background are 1 and 0, respectively.

4.

Consider a horizontal intensity profile through the middle of a binary image that contains a step edge running vertically through the center of the image. Draw what the profile would look like after the image has been blurred by an averaging mask of size $n \times n$, with coefficients equal to $1/n^2$. For simplicity, assume that the image was scaled so that its intensity levels are 0 on the left of the edge and 1 on its right. Also, assume that the size of the mask is much smaller than the image, so that image border effects are not a concern near the center of the horizontal intensity profile.

5.

The results obtained by a single pass through an image of some 2-D masks can be achieved also by two passes using 1-D masks. For example, the same result of using a 3×3 smoothing mask with coefficients $1/9$ can be obtained by a pass of the mask [1 1 1] through an image. The result of this pass is then followed by a pass of the mask

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

The final result is then scaled by $1/9$. Show that the response of Sobel masks (Fig. 10.14) can be implemented similarly by one pass of the *differencing* mask $[-1\ 0\ 1]$ (or its vertical counterpart) followed by the *smoothing* mask [1 2 1] (or its vertical counterpart).

6.

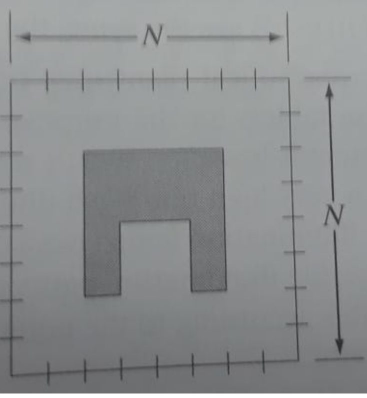
Refer to the Hough transform discussed in Section 10.2.7.

(a) Develop a general procedure for obtaining the normal representati[on of a] line from its slope-intercept form, $y = ax + b$.

(b) Find the normal representation of the line $y = -2x + 1$.

7.

Segment the image shown by using the split and merge procedure discussed in Section 10.4.2. Let $Q(R_i) = \text{TRUE}$ if all pixels in R_i have the same intensity. Show the quadtree corresponding to your segmentation.



8.

Find an expression for the signature of each of the following boundaries, and plot the signatures.

(a) An equilateral triangle

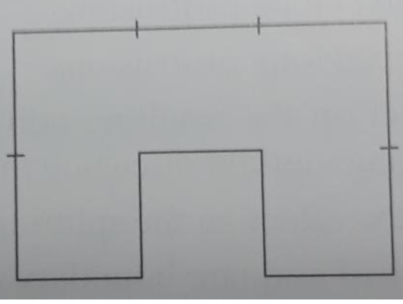
(b) A rectangle

(c) An ellipse

9.

(a) What is the order of the shape number for the figure shown?

(b) Obtain the shape number.



10.

Show that if you use only two Fourier descriptors ($u = 0$ and $u = 1$) to reconstruct a boundary with Eq. (11.2-5), the result will always be a circle.

11.

The following pattern classes have Gaussian probability density functions ω_1: $\{(0,0)^T, (2,0)^T, (2,2)^T, (0,2)^T\}$ and ω_2: $\{(4,4)^T, (6,4)^T, (6,6)^T, (4,6)^T\}$.

(a) Assume that $P(\omega_1) = P(\omega_2) = \frac{1}{2}$ and obtain the equation of the Bayes decision boundary between these two classes.

(b) Sketch the boundary.

12.

(a) Apply the perceptron algorithm to the following pattern classes: ω_1: $\{(0,0,0)^T, (1,0,0)^T, (1,0,1)^T, (1,1,0)^T\}$ and ω_2: $\{(0,0,1)^T, (0,1,1)^T, (0,1,0)^T, (1,1,1)^T\}$. Let $c = 1$, and $\mathbf{w}(1) = (-1, -2, -2, 0)^T$.

(b) Sketch the decision surface obtained in (a). Show the pattern classes and indicate the positive side of the surface.