Outline

- Representation and Description
  - Regional Descriptors
    - Basic Descriptors
    - Topological Descriptors
    - Description Using Texture
  - Description Based on Pricipal Components

Representation and Description: Basic Regional Descriptors

- Area: defined as the number of pixels in a region
- Perimeter: the length of the region's boundary
- These two are size dependent
- Compactness: perimeter^2/area

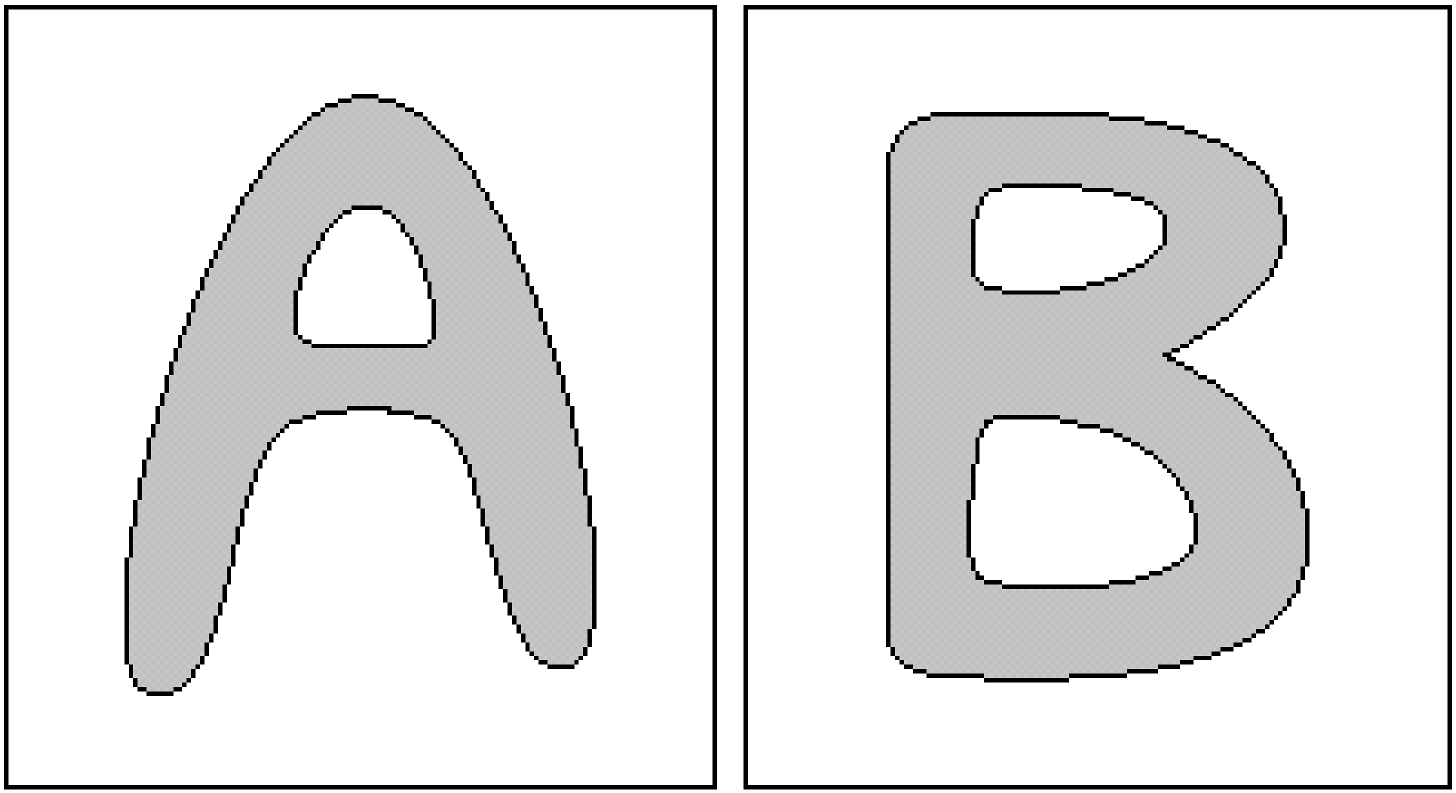
Representation and Descriptiton: Topological Region Desriptors

- Topology studies the properties that do not change with rubber-type distortions of a shape
- E.g. a cylindir and a sphere has the same topological properties
- Topological properties that can be used are: number of holes in a connected region, number of connected regions, or a combination of the the Euler number, defined as the distance between the number of connected regions and number of holes
- In case of the special polygon networks, the Euler number can be found by the following formula

$$E = V - Q + F$$

where V is the number of vertices, Q the number of edges, F the number of faces (non-hole connected subregions)

Representation and Descriptiton: Topological Region Desriptors - Example



Left: Euler number is 0, right: Euler number is -1

Representation and Descriptiton: Region Desriptors Based on Texture

- What is a texture: there is no really a definition, you can think of it as a region with similar spatial (sometimes repeating) properties
- We can use texture to describe images in two ways
  - Statistical approaches
  - Structural approaches
  - Spectral approaches

Region Descriptors Based on Texture: Statistical Approaches

- We can use statistical moments to describe textures, remember the definition of moments:

$$\mu_n(z) = \sum_{i=0}^{L-1} (z_i - m)^n p(z_i)$$

  where m is the mean and $p(z_i)$ is the probability of a pixel having the gray value z_i

- Second order moment is the variance which has information on contrast
- The third moment is a measure of the skewness of the histogram
- The fourth order moment is a measure of flatness

Region Descriptors Based on Texture: Statistical Approaches

- We can have statistical texture descriptors other than moments
- Uniformity: This is a measure of the uniformity of a texture

$$U = \sum_{i=0}^{L-1} p^2(z_i)$$

- Average entropy: This is a measure of variability, high values for images uniform histogram, 0 for constant images

$$e = -\sum_{i=0}^{L-1} p(z_i) log_2[p(z_i)]$$

Region Descriptors Based on Textures: Histogram and Spatial Information Hybrid Representations

- The moments is based on the histogram and loses all spatial information
- We can create representations that use histogram information, but also keeps some of the spatial information
- Assume that we have an image with gray levels $z_1, \ldots, z_n$, and define a position operator such as "one pixel below and one pixel right"
- Then the hybrid representation would be

$$\begin{bmatrix} a_{11} & \ldots & a_{1n} \\ \ldots & \ldots & \ldots \\ a_{n1} & \ldots & a_{nn} \end{bmatrix}$$

where a_{ij} represents the number of pixels with gray level z_i appears on "one pixel below and one pixel right" to a pixel with gray level z_j

Region Descriptors Based on Textures: Structural Approaches

- The idea behind structural approaches is that, more complicated patters can be created using simpler patterns
- Assume a represents putting a circle to the right, b to down, "c" to up, then the series "bbbaccabb" represents a 3x3 circle block

Region Descriptors Based on Textures: Spectral Approaches

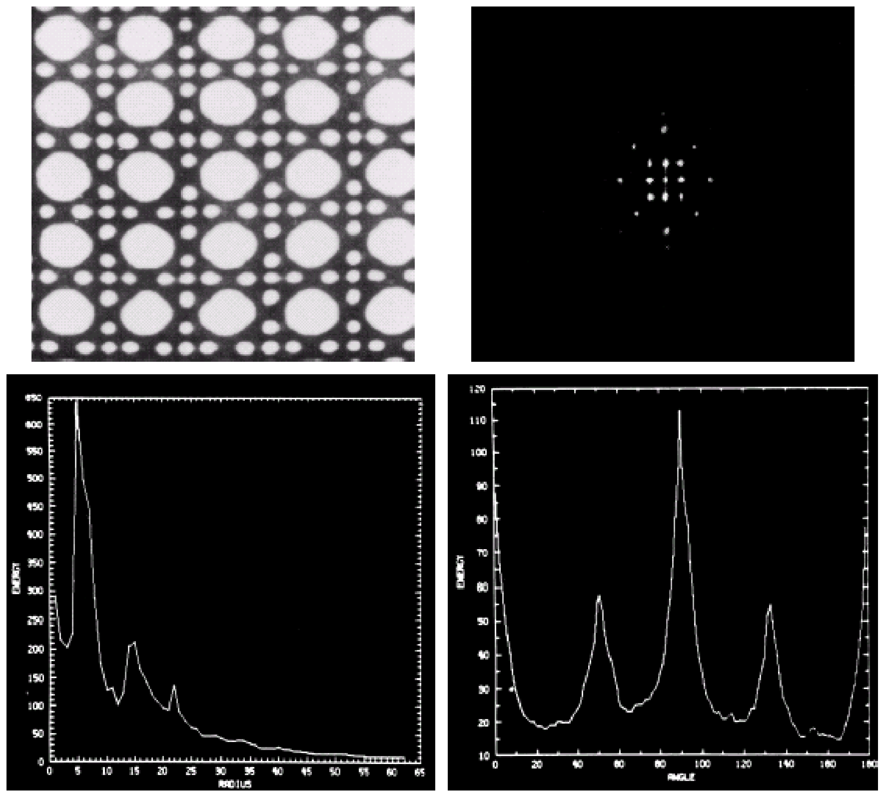
- Since textures are periodic structures, they form dominant peaks in the frequency domain
- The direction of dominant structures illustrates the direction of the structure and the location illustrates the spatial frequency
- We can represent the spectrum in polar coordinates and calculate the 1-D functions

$$S(r) = \sum_{\theta=0}^{\pi} S_\theta(r)$$

$$S(\theta) = \sum_{r=1}^{R_0} S_r(\theta)$$

- Each of these functions carry information about the direction and shape of the textures

Region Descriptors Based on Textures: Spectral Approaches - Example



Description Based on Principal Components

- Consider a vector x that has n properties of a pixel

$$x = \begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ . \\ x_n \end{bmatrix}$$

- We now have a vector representing a pixel rather than a scaler (gray value)
- All quantities such as mean, variance are now generalized to their vector and matrix counterparts such as mean vector, and the covariance matrix
- The covariance matrix C_x is a very important quantity carrying valuable information

Description Based on Principal Components

- Let us perform an eigenvalue decomposition on C_x
- Let e_i denote the eigenvectors and λ_i denote the eigenvalues
- Let us also define A as the matrix with columns that are the eigenvectors
- Hotelling transform is defined as

$$\boldsymbol{y} = A(\boldsymbol{x} - \boldsymbol{m}_x)$$

Description Based on Principal Components

- The mean and covariance of this transform can be calculated based on the mean and covariance of $\boldsymbol{x}$

$$\boldsymbol{m}_y = 0$$

$$C_y = A C_x A^{\mathrm{T}}$$

- Since A is formed by the eigenvectors of C_x C_y is diagonal:

$$C_y = \mathrm{diag}[\lambda_1, \lambda_2, \ldots, \lambda_n]$$

Description Based on Principal Components

- We can obtain $\boldsymbol{x}$ from $\boldsymbol{y}$ as follows

$$\boldsymbol{x} = A^{-1}\boldsymbol{y} + \boldsymbol{m}_x$$

- Since A is formed by eigenvectors $A^{-1} = A^{\mathrm{T}}$
- Instead of using all eigenvectors (components of A), we can truncate and use the significant ones

$$\hat{\boldsymbol{x}} = A_k^{\mathrm{T}}\boldsymbol{y} + \boldsymbol{m}_x$$

- The error between this approximation and the original value is given by

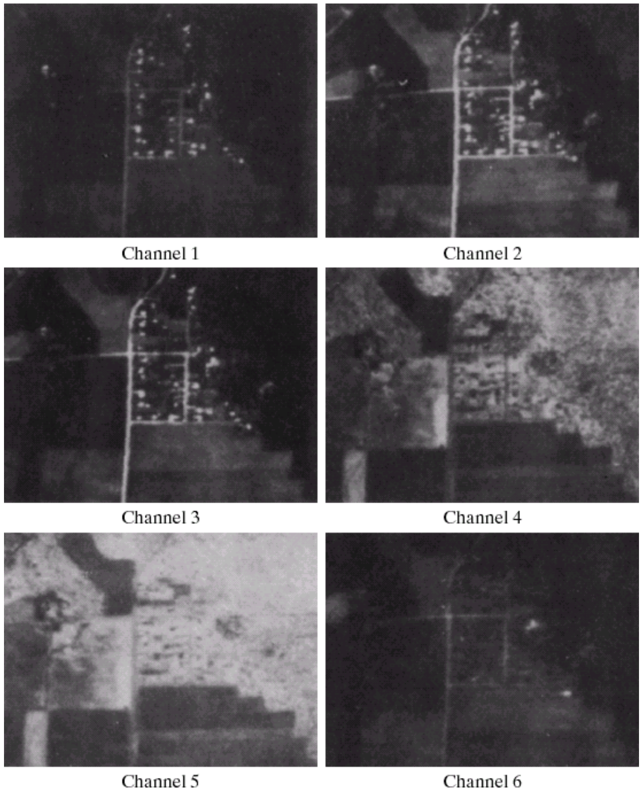
$$e = \sum_{j=k+1}^{n} \lambda_j$$

- Error is small when we ignore components only with small eigenvalues

Description Based on Principal Components

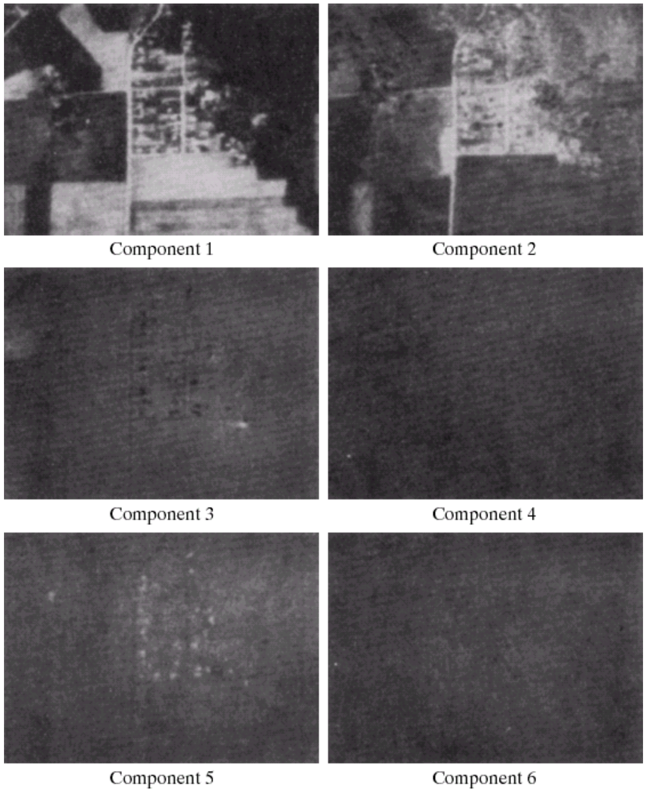
- Then for a feature vector x we can extract the principal components by eigenvalue decomposition
- The description that we can use will include only a few significant components with high eigenvalues
- Feature vector x can be obtained from one image o or multiple images of the same subject

Description Based on Principal Components - Example



Six images of the same object

Description Based on Principal Components - Example



Six principal components