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Morphological Image Processing: Introduction

- The word morphology refers to structures in the images
- We use morphological image processing to extract certain structures of the image, and processing the image before or after a certain application
- We will consider binary images first for simplicity of development
- Morphological operations are based on the set theory
- Operations such as difference, union, intersection, complement will be commonly used
- Logical operations such as and $(\cdot)$, or $(+)$, xor (x), and not $(\bar{})$ will also be used widely
- xor is defined as : $AxB = (A \cdot \bar{B}) + (\bar{A} \cdot B)$, true when A and B are different

Morphological Image Processing: Introduction

- Let us define additional operations:
  - Reflection: $\hat{A} = \{w | w = -a, \quad a \epsilon A\}$ where a is the location of a pixel
  - Translation: $(A)_z = \{w | w = a + z, \quad a \epsilon A\}$

Morphological Image Processing: Dilation Operation

- Based on these operations we define the dilation of A by B as

$$A \oplus B = \{z | (\hat{B}) \cap A \neq \emptyset\}$$

- What this equation means is that: the result of dilation operation is all translation values such that the reflected B and A overlaps
- It expands the structures in the image, hence useful for eliminating gaps, e.g. character recognition

Morphological Image Processing: Dilation Operation Example

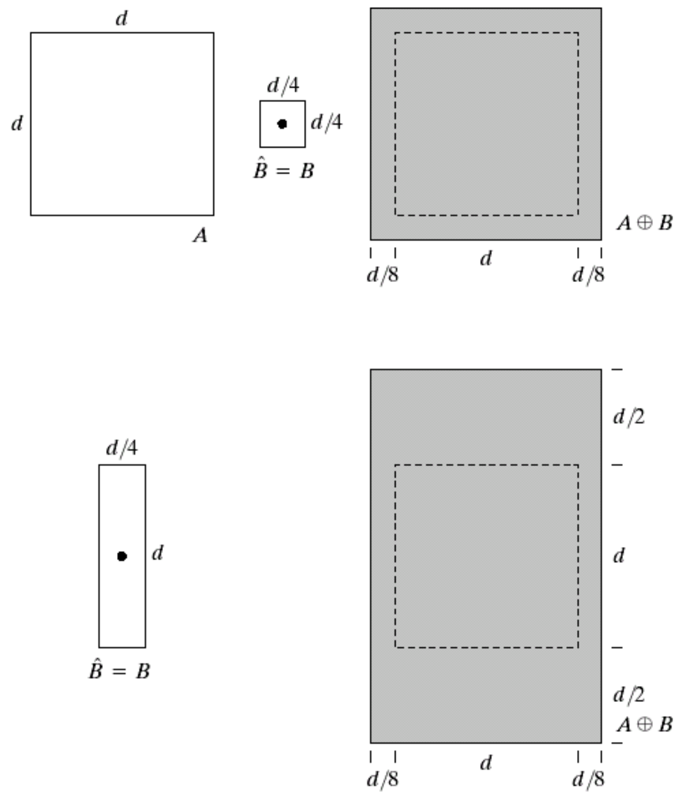
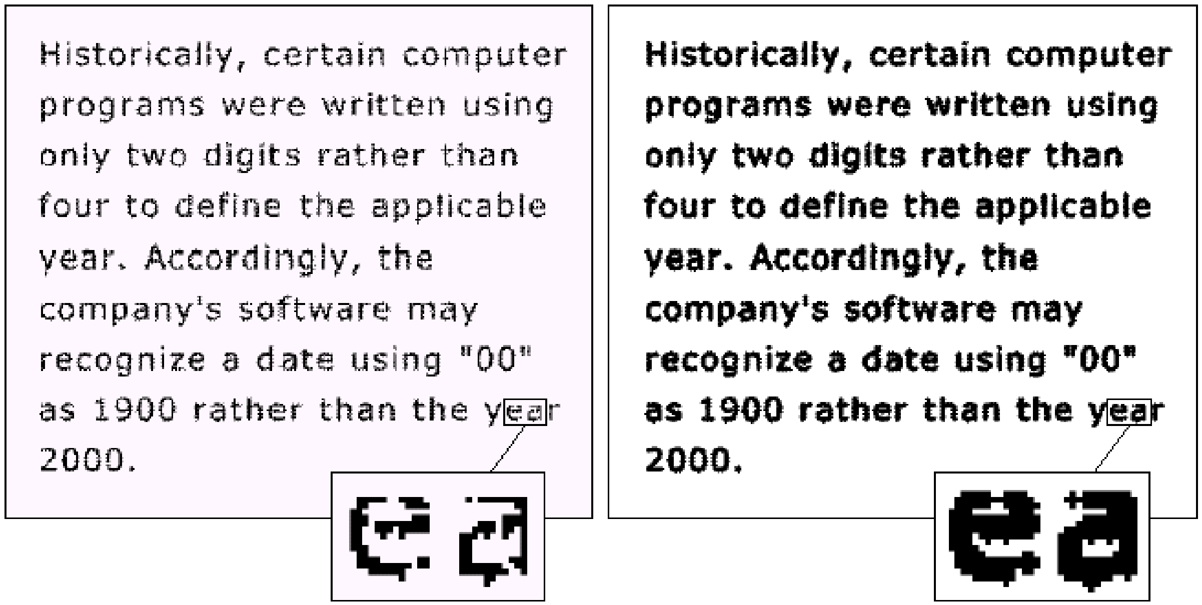


Illustration of the dilation operation

Morphological Image Processing: Dilation Operation Example



0	1	0
1	1	1
0	1	0

A real-world example of the dilation operation

Morphological Image Processing: Erosion

- Erosion of A by B is defined as

$$A \ominus B = \{z | (B)_z \subseteq A\}$$

- The result of erosion operation is then all translation values such that, translated B remains inside A
- Erosion shrinks structures in the images
- Useful for getting rid of small unwanted structures but keeping the large ones

Dilation and Erosion as Complements

- Dilation and Erosion are related by complements:

$$(A \ominus B)^c = A^c \oplus \hat{B}$$

- Easily proved using the straightforward definitions

Morphological Image Processing: Dilation Erosion Example

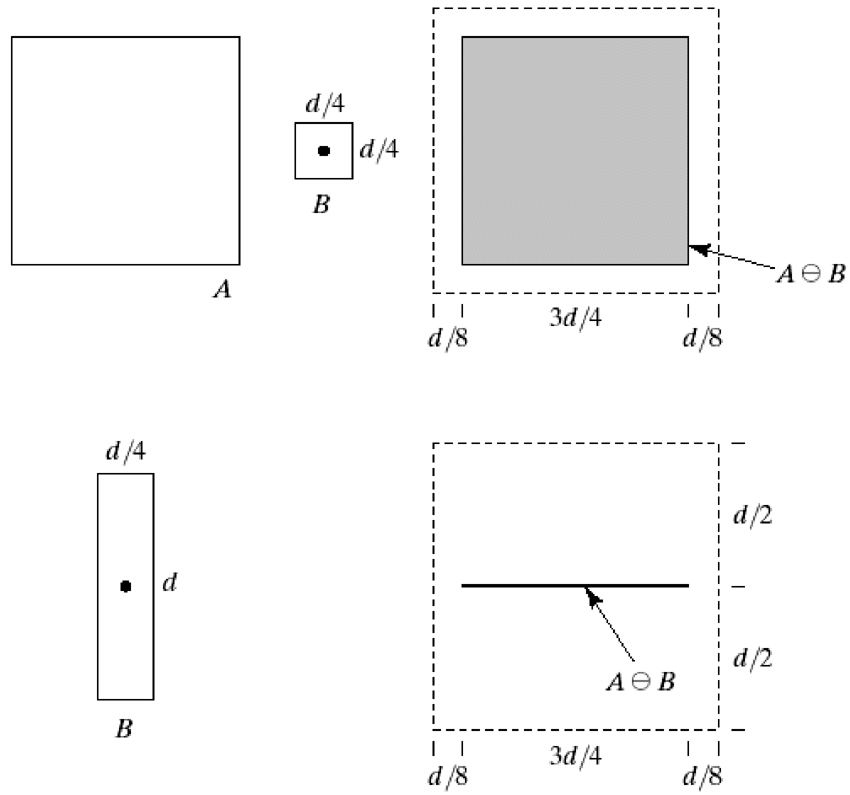
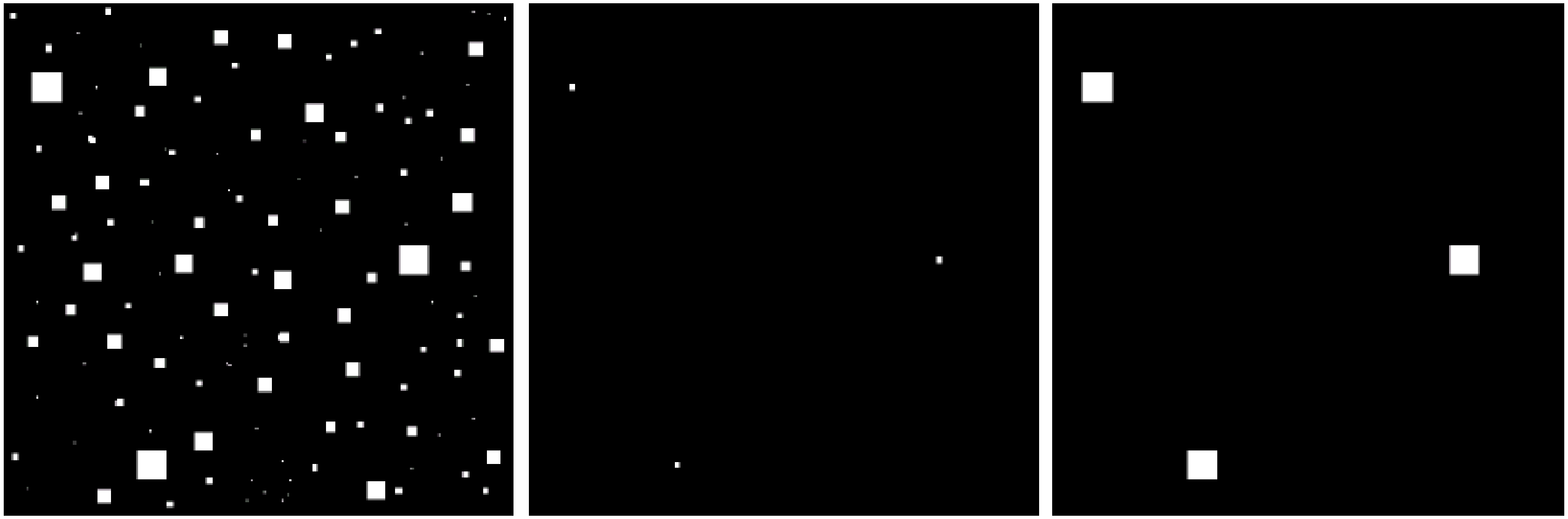


Illustration of the erosion operation

Morphological Image Processing: Dilation Erosion Example



A real-world example of the erosion operation, left: original, center: after erosion, right: after dilation of the center image

Morphological Image Processing: Opening and Closing

- Let us first see the mathematical definitions
- Opening: $A \circ B = (A \ominus B) \oplus B$
- Closing: $A \bullet B = (A \oplus B) \ominus B$
- Opening breaks thin connections since erosion is applied to the original image. The dilation that follows restores the boundaries with the openings preserved
- Closing fills small gaps since dilation is applied to the original image. The erosion that follows restores original boundary with the holes closed

Morphological Image Processing: Properties of Opening and Closing

- We have the following properties
  - $A \circ B$ is a subset of A
  - If C is a subset of D, then $C \circ B$ is a subset of $D \circ B$
  - $(A \circ B) \circ B = A \circ B$
- Similarly
  - A is a subset of $A \bullet B$
  - If C is a subset of D, then $C \bullet B$ is a subset of $D \bullet B$
  - $(A \bullet B) \bullet B = A \bullet B$

Morphological Image Processing: Opening and Closing as Complements

- Opening and closing are also related by complements:

$$(A \bullet B)^c = (A^c \circ \hat{B})$$

Morphological Image Processing: Opening and Closing Example

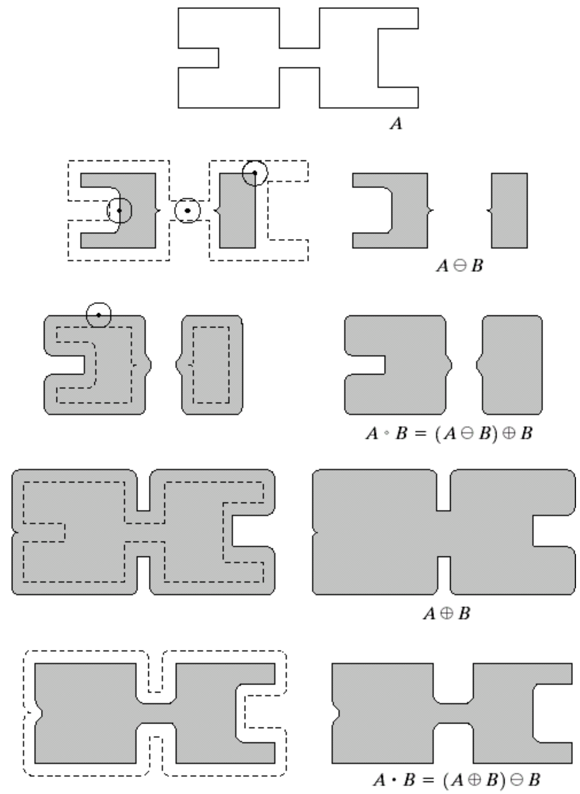
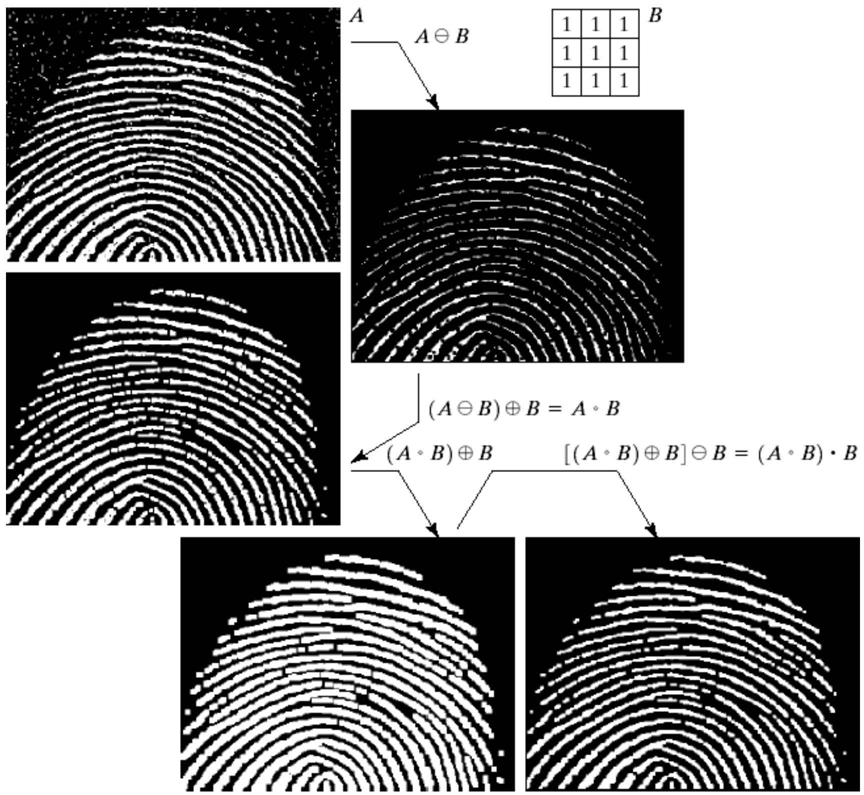


Illustration of opening and closing

Morphological Image Processing: Opening and Closing Example



A real-world example of opening and closing

Morphological Image Processing: Algorithms and Applications - Object Detection

- We can detect objects using the basic morphological operations that are explained earlier
- We need the following operation to detect an object

$$(A \ominus X) \cap [A^c \ominus (W - X)]$$

  where A is the combination of all objects, X is the object to be detected, and W is a window what includes X
- First term will have the center of X and some components from shapes larger than X, the second term will have the center of X and some components from shapes smaller than X. The intersection just have the center of X

Morphological Image Processing: Algorithms and Applications - Boundary Detection

- The boundary of an object A can be obtained in two ways
  - $A - (A \ominus B)$
  - $(A \oplus B) - A$

  where B is a small set that will determine the width if the extracted boundary
- The first boundary will be a subset of A whereas the second boundary will be one that surrounds A

Morphological Image Processing: Algorithms and Applications - Region Filling

- We know that the dilation operation expands the sets, so it can be used to fill a region with boundary A
- However, we need to stop when the dilation expands outside the boundary
- This can be achieved by taking the intersection with the original boundaries complement

$$X_k = (X_{k-1} \oplus B) \cap A^c$$

where

$$\begin{matrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{matrix}$$

Morphological Image Processing: Algorithms and Applications - Region Filling (Cont.)

- We start with X_0 a single point inside the boundary and stop when X_k equals X_{k-1}
- Then this region will grow at each iteration
- When the region grows on the boundary, the intersection with its complement will produce the same result and we stop

Morphological Image Processing: Algorithms and Applications - Extraction of Connected Components

- We can use dilation to extract connected components
- Using an iterative processing

$$X_k = (X_{k-1} \oplus B) \cap A$$

- Dilation expands the image, and the intersection with A allows to keep only pixels contained in A
- We stop when X_k equals X_{k-1}

Morphological Image Processing: Algorithms and Applications - Skeletons

- The skeleton of an object can be obtained by the following procedure

$$\cup_{k=0}^{K} S_k(A) = \cup_{k=0}^{K} (A \ominus kB) - (A \ominus kB) \circ B$$

  where $(A \ominus kB)$ denotes k successive erosions of A by B

- We stop when we reach the empty set for one iteration
- We can reconstruct the original object from its skeleton with

$$\cap_{k=0}^{K} (S_k(A) \oplus kB)$$

Morphological Image Processing: Algorithms and Applications - Extraction of Connected Components Example

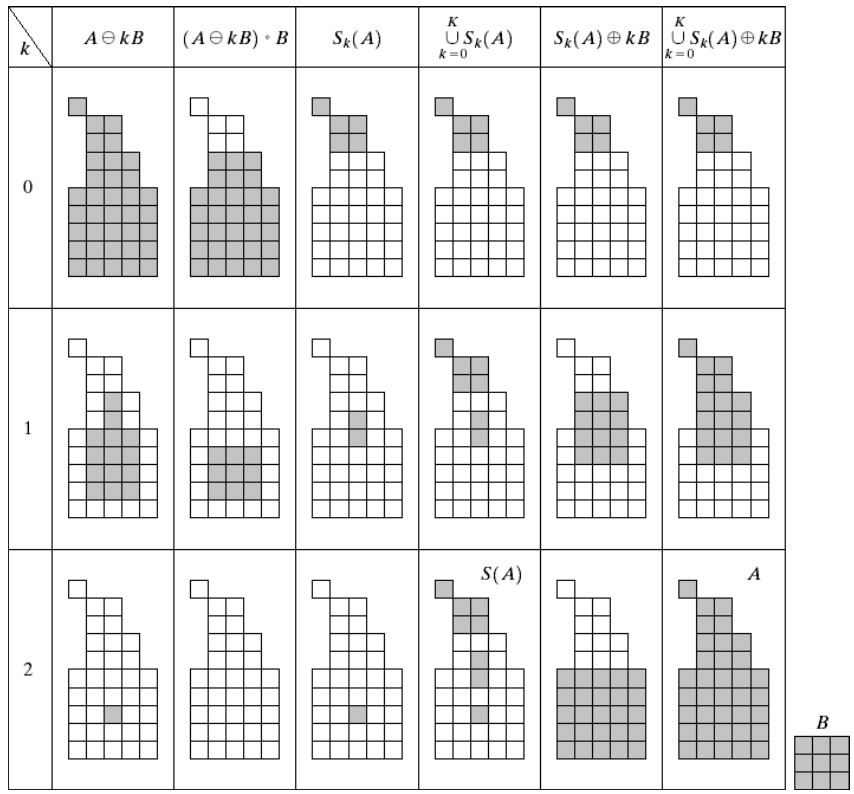


Illustration of Skeleton

Morphological Image Processing: Extensions to Gray-Scale Images

- We will show a few examples how the binary operations described can be extended to gray-scale images
- Consider an image $f(x, y)$ and a subimage (corresponding to structural element) $b(x, y)$
- Dilation for gray scale images:

  $(f \oplus b)(s, t) = \max\{f(s - x, t - y) + b(x, y) | (s - x), (t - y) \epsilon D_f; (x, y) \epsilon D_b\}$

- Erosion for gray scale images:

  $(f \oplus b)(s, t) = \max\{f(s - x, t - y) + b(x, y) | (s - x), (t - y) \epsilon D_f; (x, y) \epsilon D_b\}$

- Opening and closing can be defined based on these definitions

Morphological Image Processing: Extensions to Gray-Scale Images - Examples

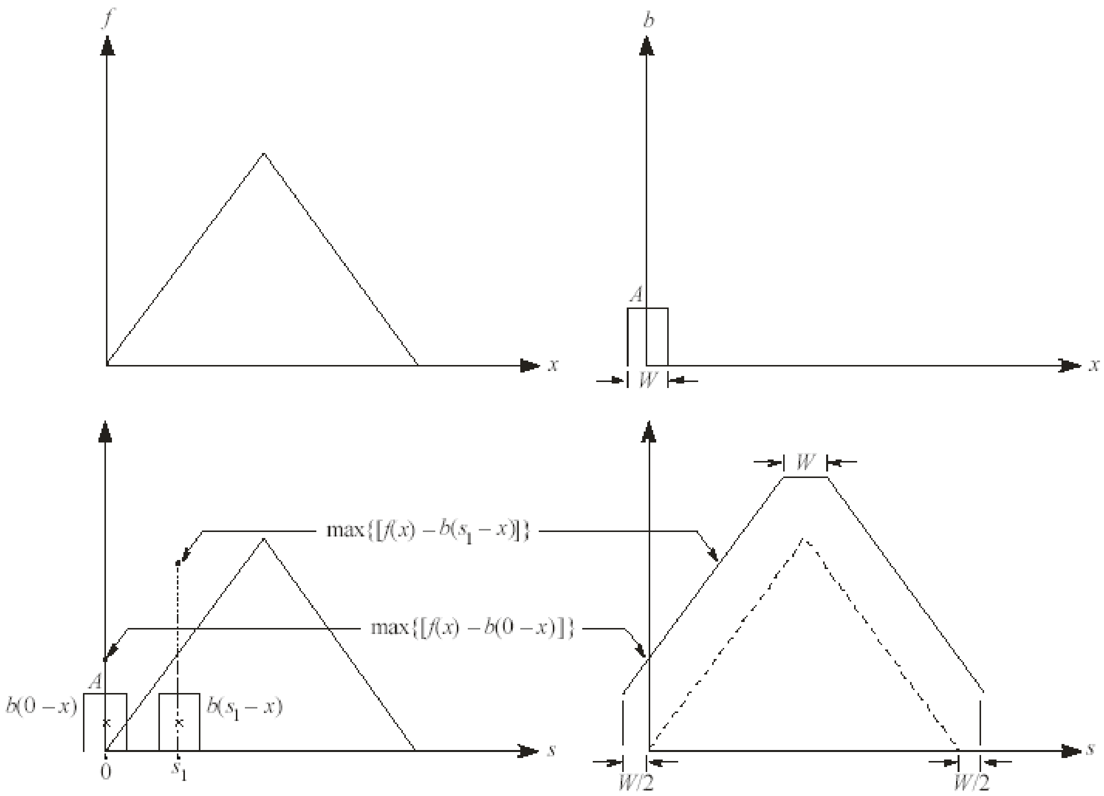
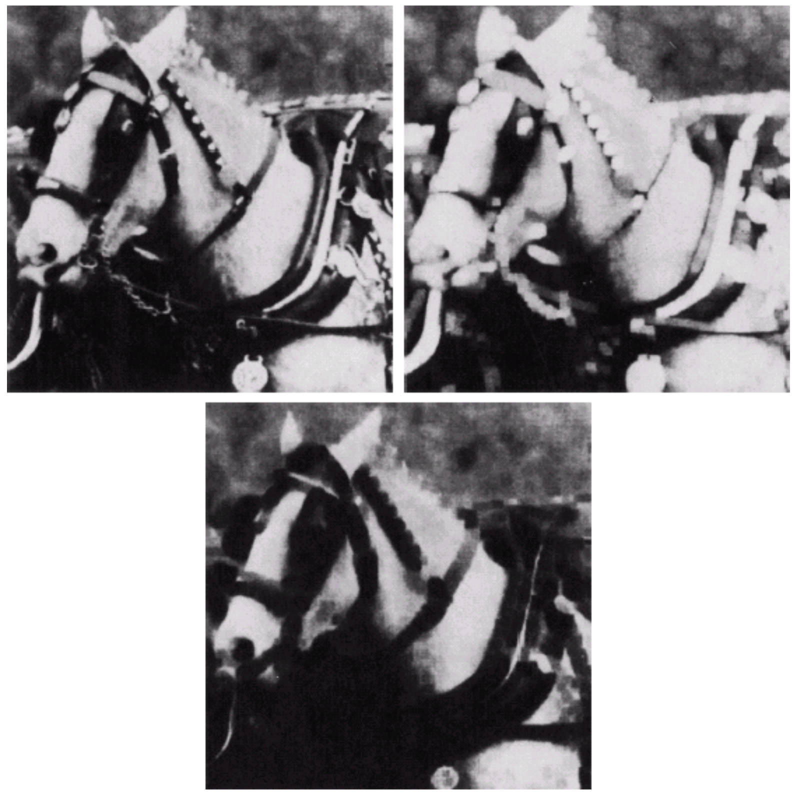


Illustration of dilation

Morphological Image Processing: Extensions to Gray-Scale Images - Examples

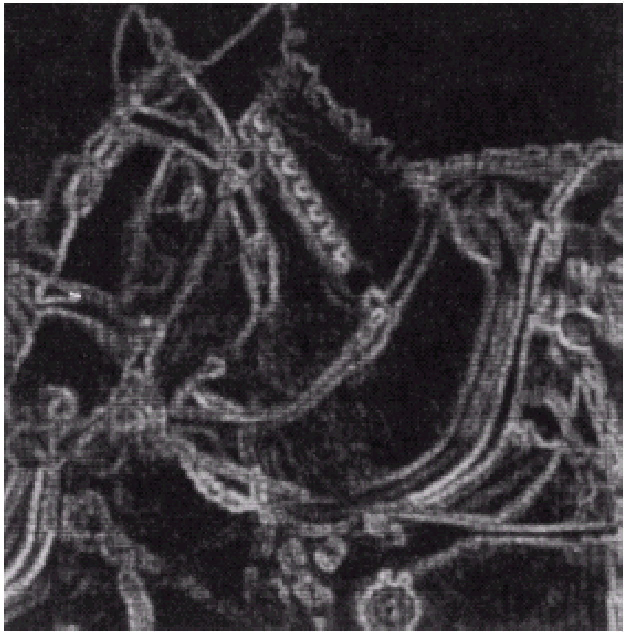


Real-world example of dilation and erosion

Morphological Image Processing: Extensions to Gray-Scale Images - Smoothing Example

Real-world example of dilation and erosion

Morphological Image Processing: Extensions to Gray-Scale Images - Gradient Example



Real-world example of dilation and erosion