Outline

- Pole-zero plots and effects on frequency response

- Digital filters introduction

- Characteristics and design parameters for digital filters

- Linear-phase FIR filters

- FIR filter design with windowing

- FIR filter design by frequency sampling

Pole-zero Plots

- Consider a system (filter) with the general frequency response

$$H(z) = \frac{\sum_{k=0}^{M} b_k e^{-jwk}}{1 + \sum_{k=1}^{N} a_k e^{-jwk}}$$

which can be written as

$$H(w) = \frac{\Pi_{k=1}^{M}(1 - z_k e^{-jwk})}{\Pi_{k=1}^{N}(1 - p_k e^{-jwk})}$$

where z_k and p_ks denote zeros and poles respectively.

- Poles and zeros determine the frequency characteristics of a filter.

- You can create a stop band by placing a zero near that frequency, or a pass-band by placing a pole near another frequency.
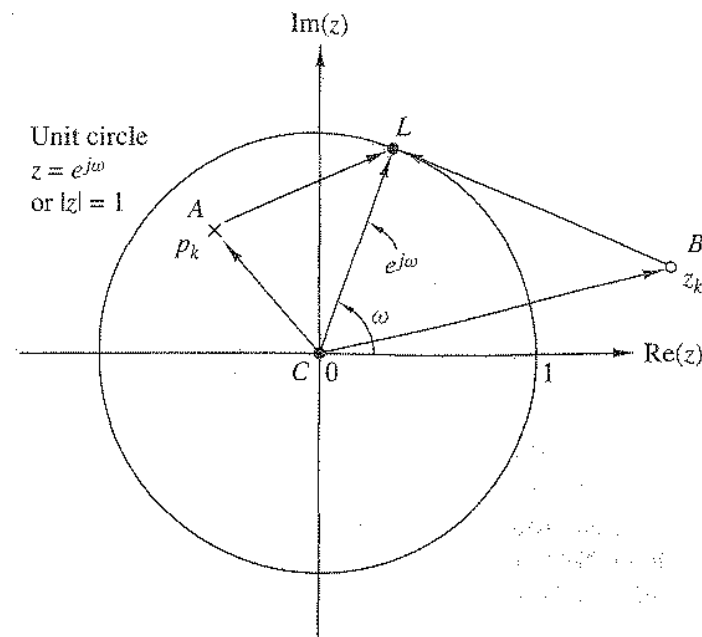
Pole-zeros and frequency response

- The magnitude response $|H(w)|$ can be written as

$$|H(w)| = \frac{V_1(w)\dots V_M(w)}{U_1(w)\dots U_N(w)}$$

  where $V(.)$ denotes distance to a zero and $U(.)$ distance to a pole

- By just looking at these distances you can calculate the magnitude response geometrically using pole-zero plot

Digital Filters Introduction

- Goal of filter design is to create a system given certain specifications regarding the frequency response

- E.g. ideal filter, perfect for passing a certain frequency range and stopping everything else

- However, is it really perfect?

- Infinitely long filters!!, not causal!!

- Then, you sacrifice certain properties to obtain realizable filters and causal filters

Digital Filters Characteristics

- Since you are restricted to realizable and sometimes causal filters, we do not have ideal filters anymore
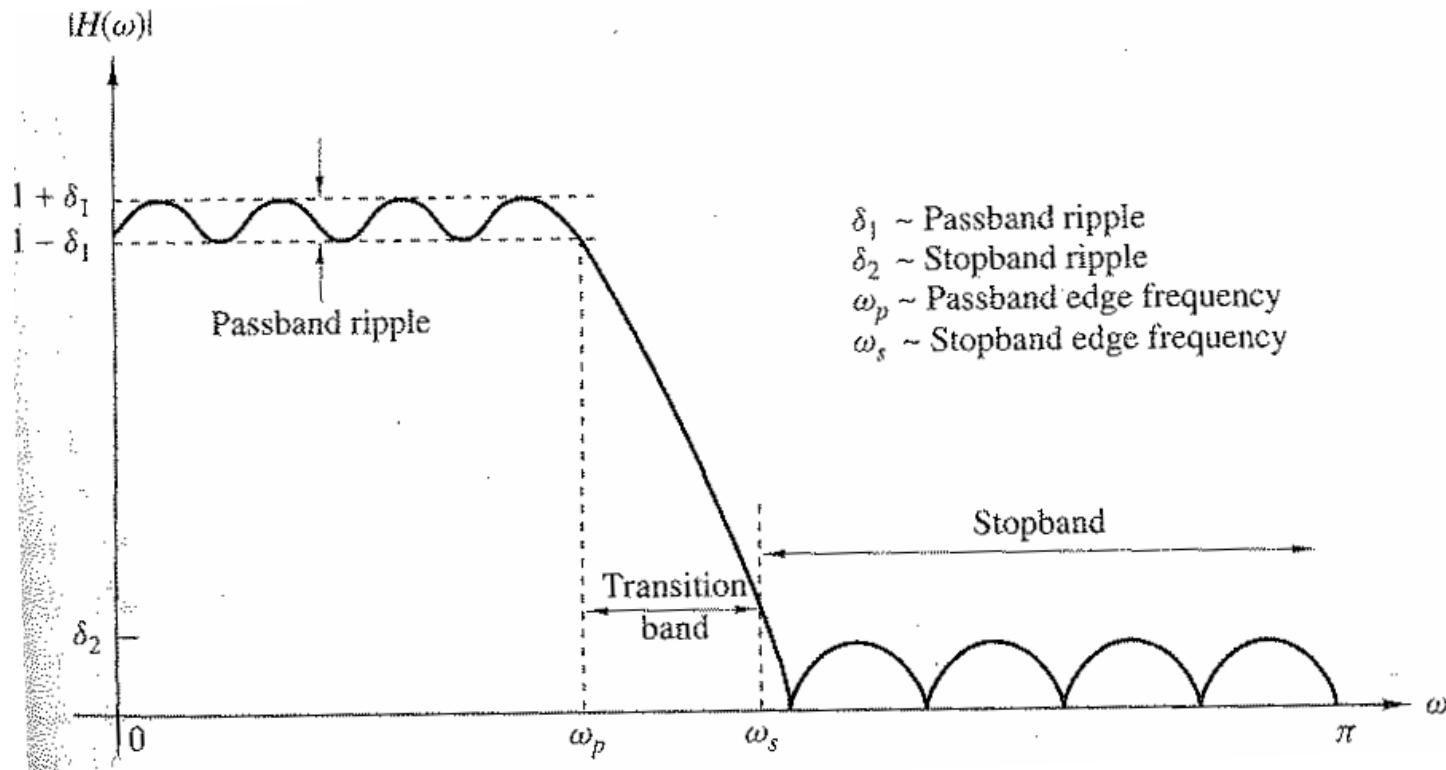
- Filter design is essentially to choose your weights (FIR or IIR) to satisfy requirements on these specifications.

Linear Phase Filters

- Consider an FIR filter

$$y(n) = b_0 x(n) + b_1 x(n-1) + \ldots + b_{M-1} X(n-M+1)$$

- The z-transform will be

$$H(z) = \sum_{k=0}^{M-1} h(k) z^{-k}$$

- A system will be linear phase if

$$h(n) = \pm h(M-1-n)$$

- With this requirement system function becomes

$$H(z) = z^{-(M-1)/2} \sum_{k=0}^{M/2-1} h(k)[z^{(M-1-2k)/2} \pm z^{-(M-1-2k)/2}$$

Linear Phase Filters

- Then if z_0 is a zero of the FIR filter, z_0^{-1} should also be a zero.

- This is a requirement for linear-phase FIR filters.

- We also know that zeros should appear in complex conjugates to create real coefficients

- Then for a complex zero z_0, there are three additional zeros $1/z_0$ (for linear phaes), z_0^* and $1/z_0^*$ (for real coefficients)

FIR Filter Design with Windowing

- Let us say we have an ideal filter that we want to reach as much as possible, denoted by $H_d(w)$ resulting in the impulse response

$$h_d[n] = \frac{1}{2\pi} \int_{-pi}^{\pi} H_d(w)e^{jwn}dw$$

- To make the filter have finite coefficients we truncate $h_d[n]$ using a window

$$h[n] = h_d[n]w[n]$$

- This multiplication in time domain will correspond to a convolution in frequency domain

$$H(w) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(\nu)W(w - \nu)d\nu$$

Linear Phase Filters

- Different windowing functions will have a different effect on the final frequency response

- Also different windowing length will similarly change the frequency response differently, the longer the window the less distortion, more costly of course

Comparison of Different Windowing Functions

TABLE 10.1 Window Functions for FIR Filter Design

Name of window	Time-domain sequence, $h(n), 0 \leq n \leq M-1$
Bartlett (triangular)	$1 - \dfrac{2\left\| n - \dfrac{M-1}{2} \right\|}{M-1}$
Blackman	$0.42 - 0.5\cos\dfrac{2\pi n}{M-1} + 0.08\cos\dfrac{4\pi n}{M-1}$
Hamming	$0.54 - 0.46\cos\dfrac{2\pi n}{M-1}$
Hanning	$\dfrac{1}{2}\left(1 - \cos\dfrac{2\pi n}{M-1}\right)$
Kaiser	$\dfrac{I_0\left[\alpha\sqrt{\left(\dfrac{M-1}{2}\right)^2 - \left(n - \dfrac{M-1}{2}\right)^2}\right]}{I_0\left[\alpha\left(\dfrac{M-1}{2}\right)\right]}$

Table 10.1 Window functions for FIR filter design
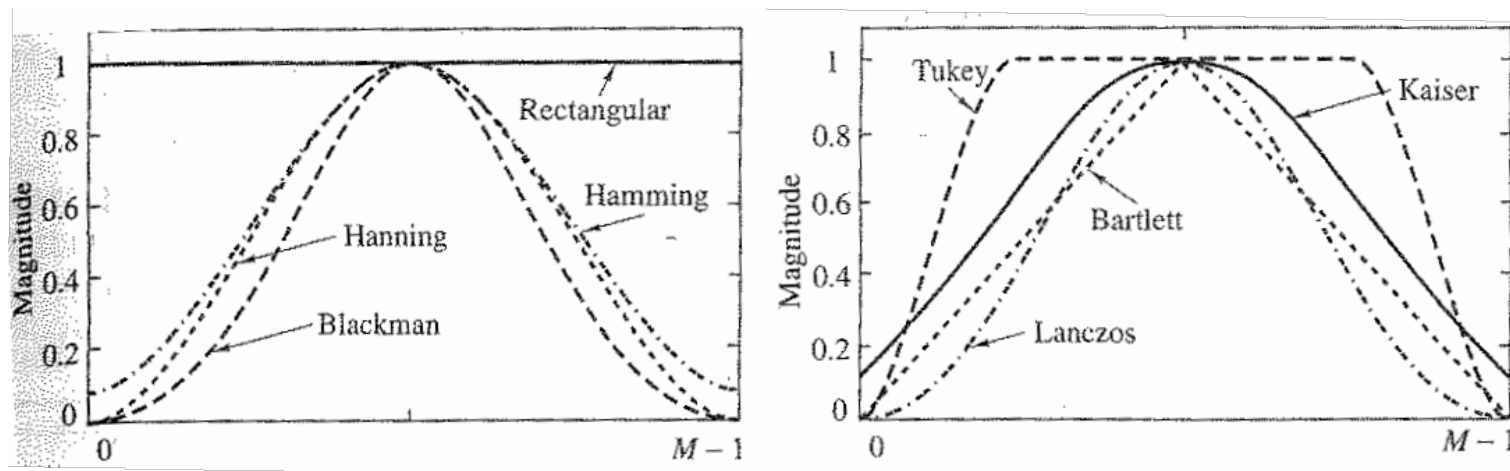
Comparison of Different Windowing Functions

Figure 10.2.3 Shapes of several windowing functions

Comparison of Different Windowing Functions

Figure 10.2.4
Frequency responses
of Hanning window
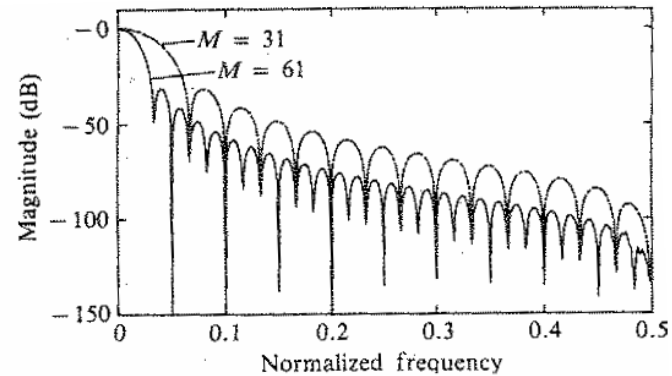for (a) $M = 31$ and
(b) $M = 61$.

Figure 10.2.5
Frequency responses
for Hamming window
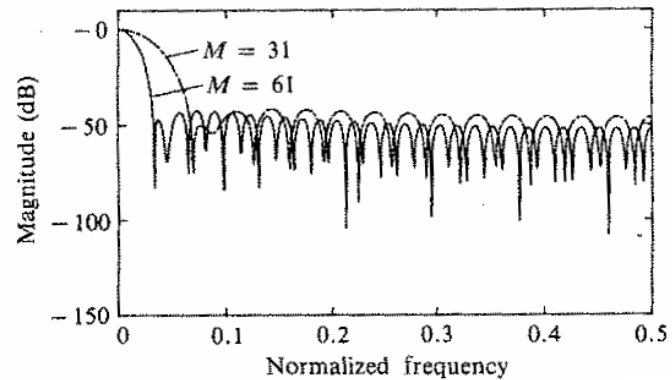for (a) $M = 31$ and
(b) $M = 61$.

Figure 10.2.4-5 Frequency responses for different windowing functions

Comparison of Different Windowing Functions

Figure 10.2.8
Lowpass FIR filter designed
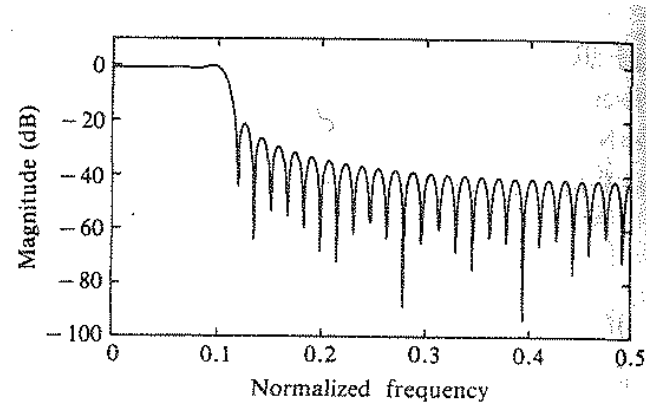with rectangular window
($M = 61$).

Figure 10.2.9
Lowpass FIR filter designed
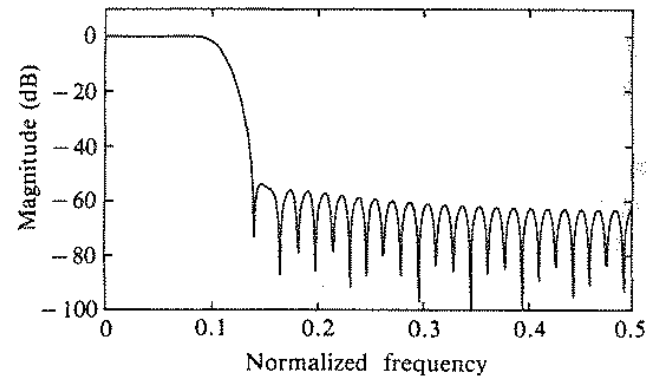with Hamming window
($M = 61$).

Figure 10.2.8-9 Frequency responses for different windowing functions

FIR Filter Design by Frequency Sampling

- In this method, we specify desired filter response at samples of $H_d(w)$ and then solve for $h_d[n]$

- Let us sample frequency at $w_k = 2\pi k/M$

- Then the samples will be

$$H_d(k) = \sum_{n=0}^{M-1} h_d[n]e^{-j2\pi nk/M}$$

- The goal is to obtain $h_d[n]$. This can be done by multiplying both sides with $e^{j2\pi nk/M}$ and summing over M samples (pretty much like calculating inverse DFT)

$$h_d[n] = \frac{1}{M} \sum_{k=0}^{M-1} H(k)e^{j2\pi kn/M}$$

- Sometimes sampling can be done at the middle of two DFT samples instead of directly the DFT samples.