Outline

- Digital Filter Banks Introduction
- Polyphase Structures for Filterbanks
- Two channel Quadrature Mirror Filterbank

Digital filterbanks introduction

- Filterbanks are used to separate the signal into its subbands in the frequency domain
- In this way signal processing can be performed for these bands individually
- There are two main parts: (i) analysis: breaks the signal into its subbands, (ii) synthesis: reconstructs the signal back from its subbands
- A filter bank is called uniform if the filters creating the subbands are identical (only shifted in frequency)

Filterbanks: general structure

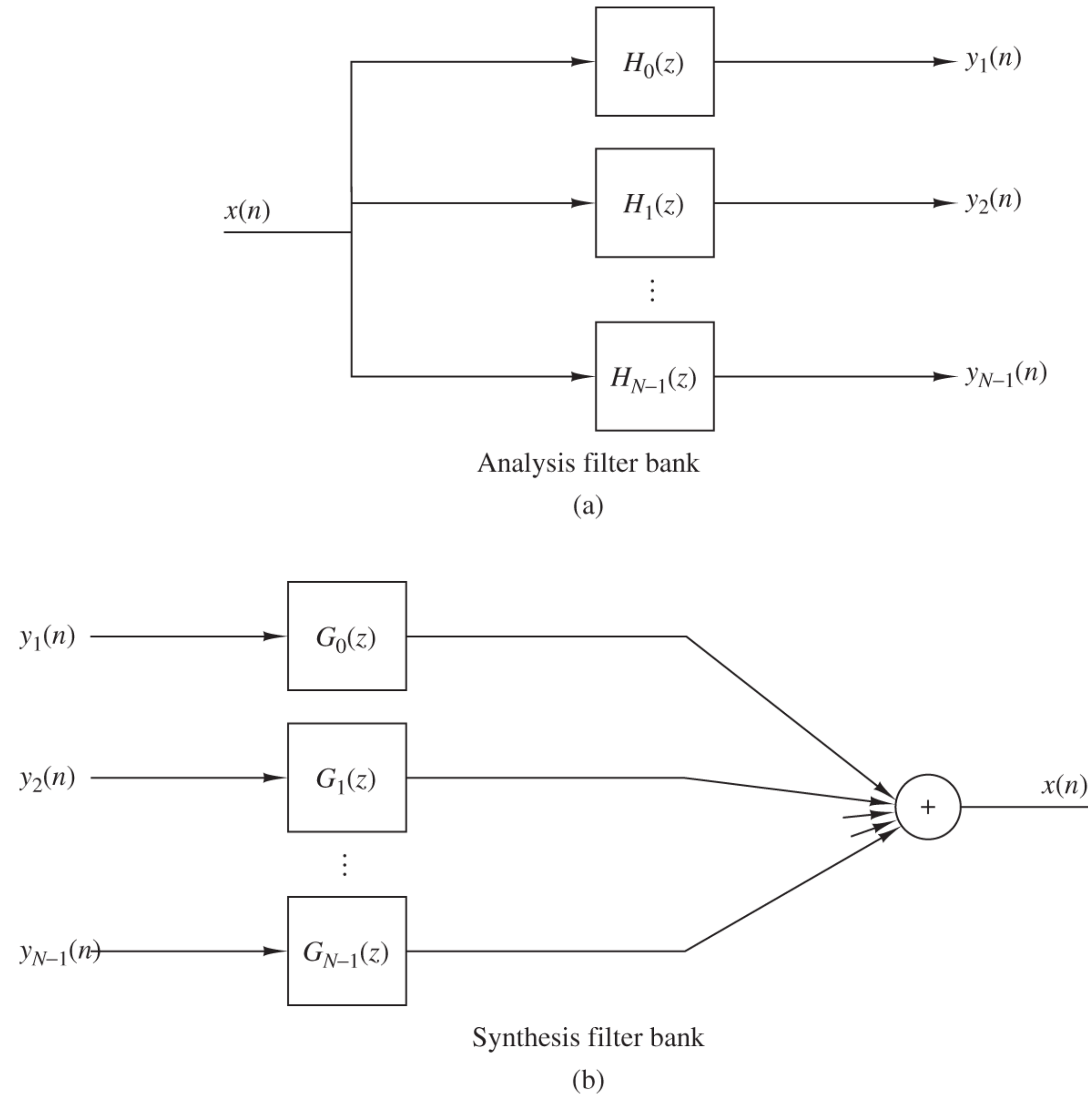

Figure 11.10.1 A digital filter bank.

Filterbanks: filters creating subbands

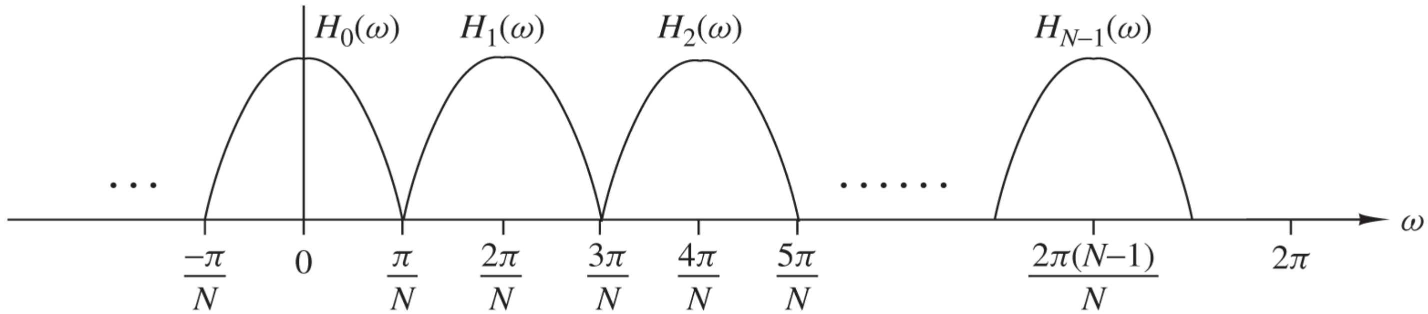

Figure 11.10.2 Illustration of frequency response characteristics of the N filters.

- The filters for a uniform filterbank can be written as:

$$h_k[n] = h_0[n]e^{j2\pi kn/N}$$

  since shifting in frequency domain is multiplication by an exponential in the time domain

- In the z-domain

$$H_k(z) = H_0(ze^{-j2\pi k/N})$$

Uniform Filterbanks Implementation

- Since the subband signals are all narrow band these can be decimated after being transferred into the lowpass region

$$x_k[m] = \sum_n h_0[mD - n]x[n]e^{-j2\pi kn/N}$$

  where k denotes the subband number and D the decimation factor which should be less than or equal to the number of subbands

- Then after processing the signals can be reconstructed back

$$v[n] = \frac{1}{N} \sum_{k=0}^{N-1} e^{j2\pi nk/N} \left[\sum_m y_k[m] g_0[n - mI] \right]$$

Uniform filterbank realization

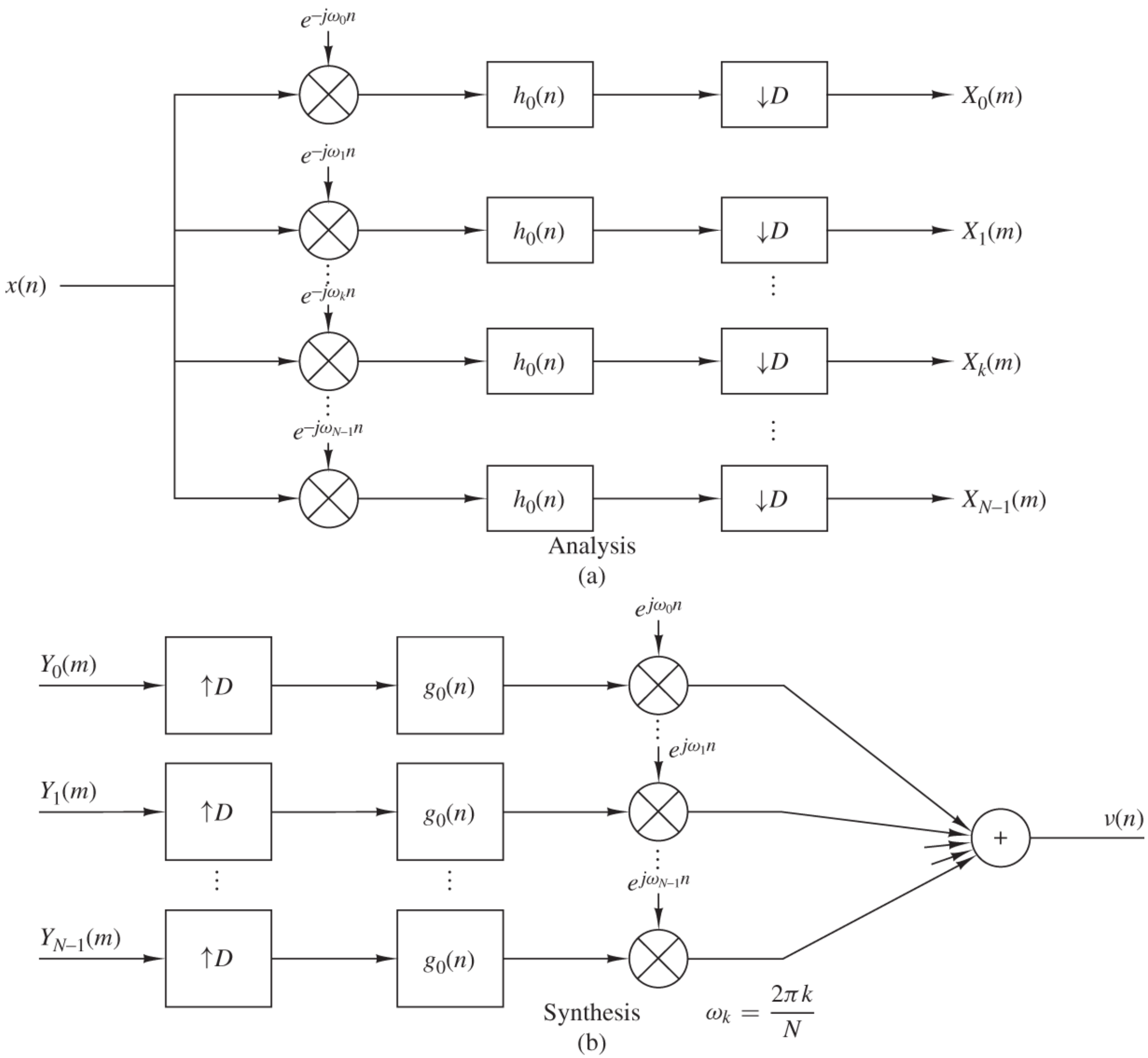

Figure 11.10.3 A uniform DFT filter bank.

Alternative uniform filterbank realization

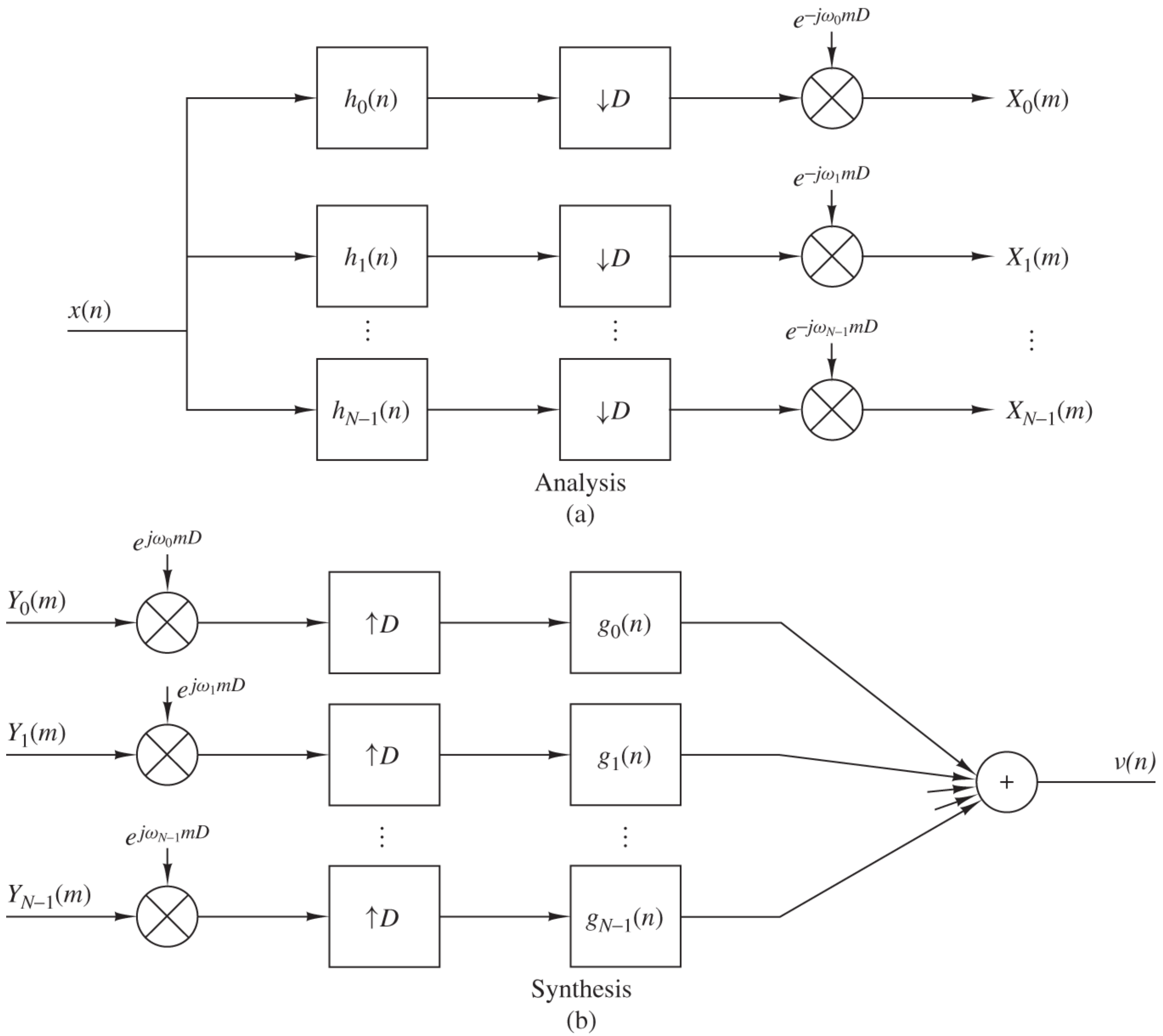

Figure 11.10.4 Alternative realization of a uniform DFT filter bank.

Polyphase implementation for uniform filterbanks

- For the analysis part, originally we have

$$x_k[m] = \sum_n h_0[mD - n]x[n]e^{-j2\pi kn/N}$$

- Using polyphase implementation

$$x_k[m] = \sum_{n=0}^{N-1} \left[\sum_l p_n[l] x_n[m-l] \right] e^{-j2\pi nk/N}$$

where $p_k[n] = h_0[nN - k]$ and $x_k[n] = x[nN + k]$

Polyphase implementation of uniform filterbanks: analysis

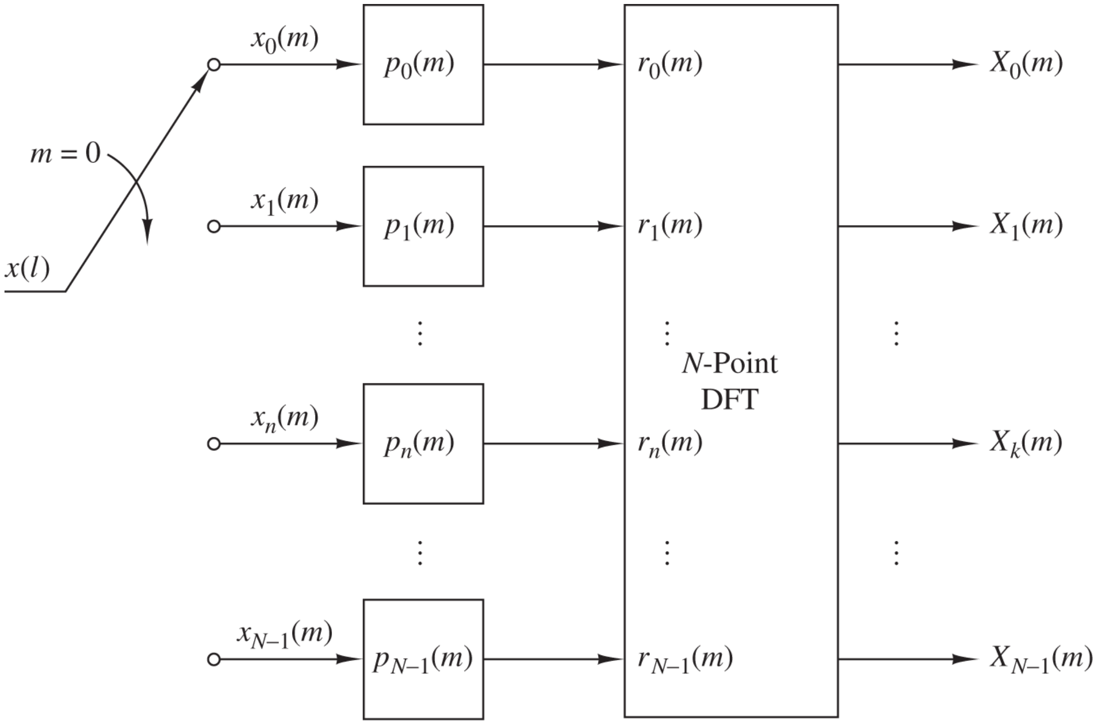

Figure 11.10.5 Digital filter bank structure for the computation of (11.10.12).

Polyphase implementation for uniform filterbanks: synthesis

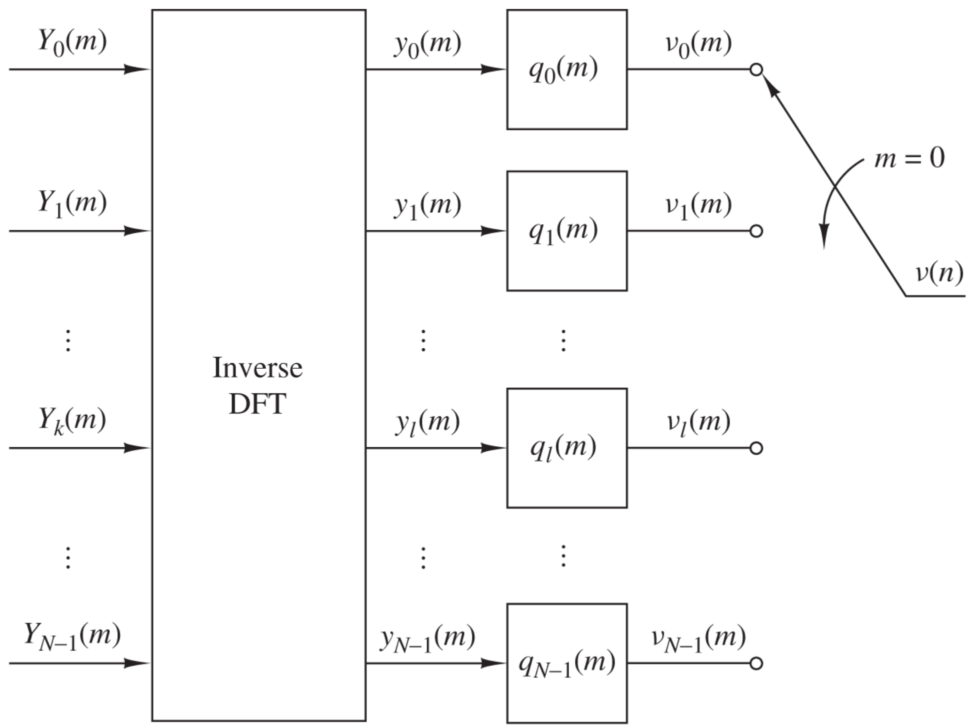

Figure 11.10.6 Digital filter bank structure for the computation of (11.10.16).

Two channel quadrature mirror filterbank

- This is an example of digital filterbanks

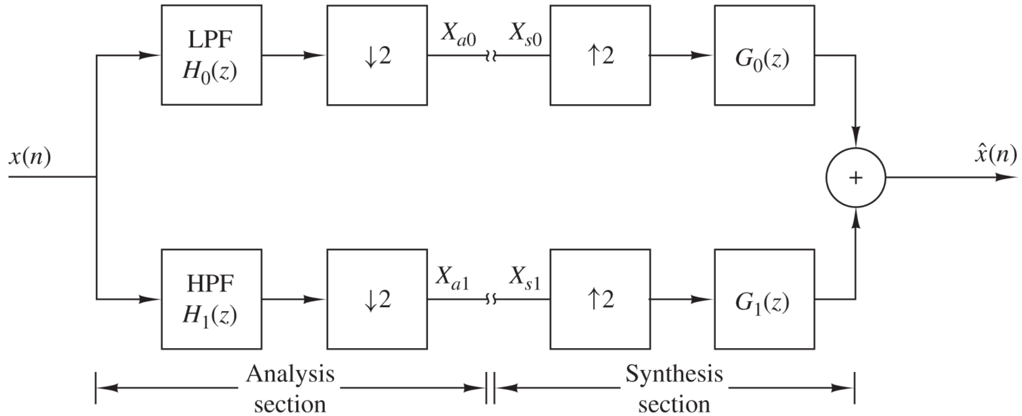

Figure 11.11.1 Two-channel QMF bank.

- We have

$$X_{a0}(w) = \frac{1}{2}\left[X\left(\frac{w}{2}\right)H_0\left(\frac{w}{2}\right) + X\left(\frac{w-2\pi}{2}\right)H_0\left(\frac{w-2\pi}{2}\right)\right]$$

$$X_{a1}(w) = \frac{1}{2}\left[X\left(\frac{w}{2}\right)H_1\left(\frac{w}{2}\right) + X\left(\frac{w-2\pi}{2}\right)H_1\left(\frac{w-2\pi}{2}\right)\right]$$

Two channel QMF Reconstruction

- The reconstructed signal is (when there is no processing in between)

$$\hat{X}(w) = X_{a0}(2w)G_0(w) + X_{a1}(2w)G_1(w)$$

resulting in

$$\begin{aligned} \hat{X}(w) &= 1/2[H_0(w)G_0(w) + H_1(w)G_1(w)]X(w) \\ &+ 1/2[H_0(w-\pi)G_0(w) + H_1(w-\pi)G_1(w)]X(w-\pi) \end{aligned}$$

- First part is the signal we want, second part is the aliasing term we do not want..

Two channel QMF Perfect Reconstruction Conditions

- We will have perfect reconstruction when aliasing term is zero:

$$[H_0(w-\pi)G_0(w) + H_1(w-\pi)G_1(w)] = 0$$

$$H_0(-z)G_0(z) + H_1(-z)G_1(z) = 0$$

- And the other term is simply a delay

$$1/2[H_0(z)G_0(z) + H_1(z)G_1(z)] = z^{-k}$$

Two channel QMF Perfect Reconstruction Conditions

- If we assume that H_1 is a mirror of H_0 (as the name implies)

$$H_0(z) = H(z), H_1(z) = H(-z)$$

- To have no aliasing

$$G_0(z) = H(z), G_1(z) = -H(-z)$$

- To make the first term exactly equal to our signal (only delays allowed)

$$1/2[H_0(z)G_0(z) + H_1(z)G_1(z)] = z^{-k}$$

  which becomes

$$H^2(z) - H^2(-z) = 2z^{-k}$$